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COSMIC STRING CATALYSIS OF SKYRMION DECAY

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ABSTRACT

We develop the Callan-Witten picture for monopole catalysed skyrmion decay in order to analyse the corresponding cosmic string scenario. We discover that cosmic strings (both ordinary and superconducting) can catalyse proton decay, but that this catalysis only occurs on the scale of the core of the string. In order to do this we have to develop a vortex model for the superconducting string. We also give an argument for the difference in the enhancement factors for monopoles and strings.



1. Introduction.

Several years ago Callan [1] and Rubakov [2] showed how it was possible for a grand unified monopole to catalyse proton decay. They also showed that the catalysis would occur with an enhanced cross-section: the inverse square of the proton rather than the grand unified mass scale. That a grand unified topological defect can catalyse baryon decay is not surprising. In such objects, the symmetry of the grand unified group (e.g. $SU(5)$ or $SO(10)$) is unbroken in the core of the defect, thus allowing baryon non-conserving reactions to occur. What is somewhat surprising is the enhancement of the cross-section. Naively we might expect that only a 'direct hit' of the proton on the monopole core would allow such a catalysis to proceed, however, due to the magnetic moment of the fermion there is an attractive force binding it to the monopole - thus leading to the enhancement factor.

The cosmological implications of this result are immediate. If topological defects exist, then they can catalyse baryon decay. If they catalyse baryon decay at a sufficient rate then the effects should be observable. The lack of observational data to support proton decay at such rates then places a constraint on the number density of such defects. Thus the enhancement factor in the case of monopoles actually places more stringent bounds on the monopole flux [3,4] than conventional methods [5].

Cosmic strings are also topological defects of grand unified theories. Thus, like monopoles, they can catalyse baryon decay. However, it has recently been shown that for both cosmic [6] and superconducting cosmic strings [7], there is no enhancement of the cross-section; the catalysis cross-section is just given by the grand unified scale. Since the density of strings is very high at early times in the evolution of the universe, it is still possible that string catalysis could have cosmological implications despite the smallness of the cross-section. For example, string catalysis could place constraints on baryogenesis [8].

It is therefore important that monopole and string catalysis are fully understood. The quark-monopole(string) scattering is fairly well understood, but it describes a high energy process. In low energy processes (such as decay) we should consider a more appropriate hadron model. In the cosmological context, where we expect fairly low velocities, it is

important that a more appropriate model is developed. One such model is the Skyrme model [9], which has the advantage over hadronic models in that it is easy to couple to strings or monopoles. A more realistic low-energy hadron model, such as the skyrmion, might yield some additional insight into the physical decay process. Callan and Witten [10] were the first to examine such a setup. They considered a skyrmion model for the hadron, a Dirac monopole, and examined the processes occurring during the skyrmion-monopole interaction.

We use this as a starting point for our discussion. In section 2 we give a brief résumé of the Skyrme model. We review and develop the Callan-Witten [10] argument in section 3 using the Wu-Yang [11] picture of a monopole. Such a picture removes the problem of the distinction between the physical singularity of the electromagnetic fields at the monopole core and the Dirac string singularity, which is a gauge artefact. Our argument also clarifies aspects of the topological unwinding of the skyrmion on the monopole. In the fourth section we examine the scattering of a skyrmion off a cosmic string. We first use the wire model for the string in order to mimic the Dirac model for the monopole, however such a picture does not permit baryon decay. We are therefore forced to consider a vortex model for the string in order to obtain catalysis in the string core. In section 5 we consider the analogous process for a superconducting string [13]. First we use the wire model, but despite there being a long range force in this case, we again show that such a picture does not result in baryon decay. We then develop a vortex model for the superconducting string, by solving the string equations of motion near the core. In the vortex model we obtain catalysis in the string core. Our analysis gives a heuristic explanation of the enhancement factor with monopoles, which we explain in the concluding section.

2. Résumé of the Skyrme model.

In this section we review the Skyrme model of the nucleon [9]. This is a sigma model with stable soliton solutions otherwise known as skyrmions. (For a good review see e.g. Balachandran [14].) The lagrangian is

$$\mathcal{L}_{SK} = \frac{1}{16} f_\pi^2 \text{Tr} \partial_\mu U \partial^\mu U^{-1} + \frac{1}{32a^2} \text{Tr} [U^{-1} \partial_\mu U, U^{-1} \partial_\nu U]^2, \quad (2.1)$$

where a is a constant of order one. Here U is an $SU(2)$ matrix related to the pion fields via

$$U = \exp \left\{ \frac{2i}{f_\pi} \underline{\tau} \cdot \underline{\pi} \right\}, \quad (2.2)$$

where $\underline{\tau} = (\tau_1, \tau_2, \tau_3)$ are the three generators of $SU(2)$. An alternative representation of U , which will be useful in what follows, is one which makes explicit reference to the S^3 topology of $SU(2)$:

$$U = \cos \frac{2|\underline{\pi}|}{f_\pi} I_2 + i \sin \frac{2|\underline{\pi}|}{f_\pi} \underline{\tau} \cdot \hat{\underline{\pi}}. \quad (2.3)$$

U is now represented as a unit four-vector in terms of the basis $\{I_2, \underline{\tau}\}$, where I_2 is the identity matrix.

The lagrangian (2.1) admits soliton solutions, i.e. localised stable finite energy field configurations. A topological picture of this can be drawn in the following way. Finiteness of the energy requires that $U(\underline{x}) \rightarrow \text{const.}$ as $|\underline{x}| \rightarrow \infty$. We can therefore think of a soliton field configuration as a map from compactified three-space ($\mathbb{R}^3 \cup \{\infty\} \cong S^3$) to the three-sphere of $SU(2)$,

$$U_{\text{sol}}(\underline{x}) : S_{\text{phys}}^3 \rightarrow S_{\text{SU}(2)}^3. \quad (2.4)$$

Such maps may be classified according to the homotopy equivalence class to which they belong. Members of the same equivalence class are related to each other by a

continuous deformation and represent translated or excited states of the same soliton. Since $\Pi_3(S^3) \cong \mathbb{Z}$, we may conclude that soliton field configurations are labelled uniquely by an integer value, N_B (the baryon number), which is the degree of the map (2.4). In a dynamical theory, the continuity of the fields implies that N_B is a continuous function of time and hence constant.

The baryon number may be alternately represented as the charge associated with the conserved current

$$B_s^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left(U^{-1} \partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U \right) \quad (2.5a)$$

$$N_B = \int B^0 d^3 x . \quad (2.5b)$$

The standard nucleon field configuration ($N_B = 1$) is usually represented by the Hedgehog configuration:

$$U_N(\underline{x}) = \exp[iF(r)\hat{x} \cdot \underline{\tau}] , \quad (2.6)$$

where $F(0) = \pi$ and $F(\infty) = 0$. In practise $F(r)$ differs significantly from zero only within the core of the skyrmion, i.e. a distance $\sim (af_\pi)^{-1}$.

In the presence of electromagnetism, the preceding discussion must be modified to allow for the nucleon charge and magnetic moment interaction. We must generalise \mathcal{L}_{SK} to be invariant under

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha \quad (2.7a)$$

$$U \rightarrow U + ie\alpha[Q, U] \quad (2.7b)$$

where Q is the quark charge matrix:

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} . \quad (2.8)$$

(For simplicity we are considering the case of two quark flavours only.)

Taking into account QCD anomalies, the lagrangian and baryon current become, [15],

$$\begin{aligned}
\mathcal{L} = & \frac{1}{16} f_\pi^2 \text{Tr} \mathcal{D}_\mu U \mathcal{D}^\mu U^{-1} + \frac{1}{32a^2} \text{Tr} [U^{-1} \mathcal{D}_\mu U, U^{-1} \mathcal{D}_\nu U]^2 \\
& + \frac{e}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} A_\mu \text{Tr} Q [\partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U U^{-1} \\
& \quad + U^{-1} \partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U] \\
& + \frac{ie^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu A_\nu) A_\rho \text{Tr} [Q^2 \partial_\sigma U U^{-1} + Q^2 U^{-1} \partial_\sigma U \\
& \quad + \frac{1}{2} Q \partial_\sigma U Q U^{-1} - \frac{1}{2} Q U Q \partial_\sigma U^{-1}] \quad (2.9a)
\end{aligned}$$

$$B^\mu = B_s^\mu + \frac{ie}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu [A_\rho \text{Tr} Q (U^{-1} \partial_\sigma U + \partial_\sigma U U^{-1})], \quad (2.9b)$$

where $\mathcal{D}_\mu U = \partial_\mu U - ie A_\mu [Q, U]$ is the covariant derivative of U .

Note that (2.7b) is infinitesimal in form. We will need the non-infinitesimal version:

$$U \rightarrow e^{ie\alpha(z)Q} U e^{-ie\alpha(z)Q}.$$

Noting that $Q = \frac{1}{6} I_3 + \frac{1}{2} \tau_3$, we see that this reduces to

$$U \rightarrow e^{ie\alpha(z)\tau_3/2} U e^{-ie\alpha(z)\tau_3/2}. \quad (2.10)$$

Note that the generalisation of the baryon current (2.9b) now contains a term dependent on A_μ . This appears via a term which is a divergence, thus provided there are no singularities in A_μ , and that surface terms vanish, the baryon number is still integral. In terms of the topological picture presented previously, provided there are no singularities, $U_{\text{sol}}(x)$ is still a map from $S^3 \rightarrow S^3$ and thus the classification of maps into equivalence classes labelled by baryon number still holds.

3. Monopole catalysed skyrmion decay.

In this section we review the argument of Callan and Witten [10] for skyrmion decay. Our presentation is from a different viewpoint in order to facilitate the transition to the string picture. For simplicity let us now consider the Dirac monopole. In its original formulation this has a gauge potential given by[†]

$$A_\phi = g(1 - \cos \theta) . \quad (3.1)$$

In the ensuing discussion, we will assume that $eg = 1/2$, in accordance with the Dirac quantisation condition. This is singular on the line $\theta = \pi$, however the electromagnetic flux,

$$F_{\theta\phi} = g \sin \theta, \quad (3.2)$$

is finite everywhere except at $r = 0$. Thus the singularity of A_μ on $\theta = \pi$ is a gauge artefact, the Dirac string. It arises because we are trying to express the electromagnetic field tensor as the exact differential of a covector gauge field on $\mathbb{R}^3 - \{0\}$. Since the gauge field now does contain singularities, the interpretation of the divergence in the second term of (2.9b) is unclear - is the baryon number still integral? The mathematical/topological picture also becomes clouded. If one removes the semi-infinite singular line $\{\theta = \pi\}$ from \mathbb{R}^3 , the soliton field configurations are now maps from $\mathbb{R}^3 - \{\theta = \pi\}$ (which is contractible) into S^3 . These are all topologically trivial. Clearly this is too naive, for the field U must satisfy some continuity property near $\theta = \pi$ in order for it to contain no singularities. However, since U is coupled to the gauge field A_μ , we must be careful in specifying boundary conditions along $\theta = \pi$.

In order to make these intricacies more transparent, we will take an approach to the Dirac monopole which avoids Dirac strings - that due to Wu and Yang [11]. Briefly, the singularity in A_μ is exactly analogous to the coordinate singularity we obtain if we try to cover S^2 with only one coordinate patch. Just as this singularity can be removed by

[†] Note that here, and throughout, we use a coordinate basis for our components and not an orthonormal one.

choosing two patches, the singularity in A_μ can be removed if one chooses instead two coordinate patches for $\mathbb{R}^3 - \{0\}$, each with an associated A_μ , relating the two different 'branches' of A_μ by a gauge transformation on the overlap.

Two convenient patches are

$$(1) \quad 0 \leq \theta < \pi - \delta \quad ; \quad r > 0 \quad (3.3a)$$

$$(2) \quad \delta < \theta \leq \pi \quad ; \quad r > 0, \quad (3.3b)$$

with

$$A_{1\phi} = g(1 - \cos \theta) \quad (3.4a)$$

$$A_{2\phi} = -g(1 + \cos \theta). \quad (3.4b)$$

These are related by the non-trivial gauge transformation

$$A_{2\mu} = A_{1\mu} - 2g\partial_\mu\phi \quad (3.5)$$

on the overlap. The Dirac string picture is the limit as $\delta \rightarrow 0$ of one coordinate patch. We see that this picture now has no coordinate singularities. The gauge field on each coordinate patch is perfectly regular. The electromagnetic flux is $g \sin \theta$ on each patch and is independent of the branch of A_μ chosen on the overlap. This is what we would expect for a physical quantity.

We now want to include the $SU(2)$ field, U , in this picture. The nucleon field is non-singular, therefore it must be well defined everywhere. Since the U -field is coupled to the gauge field, the presence of the two branches of A_μ indicates that we must define a separate field configuration on each chart. These will then be related in the overlap by a non-trivial transformation induced by the gauge transformation (3.5) on A_μ . From (2.10) we conclude that this is

$$U_2 = e^{-i\phi\tau_3/2} U_1 e^{i\phi\tau_3/2}. \quad (3.6)$$

We now have a perfectly consistent, singularity free picture of the nucleon on the background field of the monopole.

Having removed the singularity problem, we see that once again the $SU(2)$ field configuration is a map from compactified physical space into the $SU(2)$ three-sphere. However,

whereas before we could regard \mathbb{R}^3 as the union of the two coordinate patches (1) and (2) with the trivial gauge transformation on U , here we have a non-trivial transformation for U in the overlap. Thus although we can classify the field configurations in each case according to homotopy equivalence, there is no reason to assume that in each case these classes will be the same. Indeed, if we write U in the form (2.2), the effect of the gauge transformation (2.10) is to rotate the vector \hat{n} by an angle ϕ around the 3-axis. This will have a twisting effect on the π_1, π_2 components. Thus the presence of the monopole gauge field shuffles the members of the equivalence classes of (2.4). This 'shuffling' is crucial to the physical description which follows.

For convenience and comparison with the cosmic string case we will subdivide the argument into three steps: firstly we show that there is a suppression of the wave function of a charged particle near the monopole core; secondly, that due to the shuffling of the baryon equivalence classes it is possible for a pure π^0 field configuration to carry baryon number; and thirdly, that under the presence of suitable boundary conditions there can be a non-zero radial baryon flux — the skyrmion unwinds.

i) Suppression of charged particles near core.

Solving the Klein-Gordon equation,

$$(\nabla_\mu \nabla^\mu - e^2 A_\mu^2)\varphi = 0, \quad (3.7)$$

for the charged pions in the presence of a magnetic monopole shows that charged pions can no longer have zero angular momentum. Instead of the usual spherical harmonics, we must now use generalised monopole harmonics [12] for the wave function in the monopole background. These have a dependence on the charge of the particle. The lowest harmonic for the patch (1) is $\varphi \propto \sqrt{1 + \cos\theta}$. Substituting this into (3.7) gives for the radial part of the wave equation:

$$\frac{1}{r^2} \partial_r r^2 \partial_r \varphi = \frac{1}{2r^2} \varphi.$$

This implies an asymptotic behaviour of $\varphi \propto r^{\frac{\sqrt{8}-1}{2}}$ as $r \rightarrow 0$. Thus the wave function of the charged pions is suppressed near the core. Note however that for uncharged particles

no such suppression occurs. Thus in order for the nucleon to approach the monopole core, it must be able to deform into a pure π^0 field configuration. In order for this process to be possible, the π^0 field configuration must be able to carry baryon number.

ii) *Baryon number of a π^0 configuration.*

For simplicity we consider a purely radial π^0 field configuration

$$\begin{aligned} U_K &= \exp\{if\tau_3\} \\ &= \cos f + i \sin f \tau_3, \end{aligned} \quad (3.8)$$

where $f(0) = 0$ and $f(\infty) = 2\pi$. It is then straightforward to calculate the baryon number from (2.9b):

$$\begin{aligned} B_\mu &= \frac{1}{24\pi^2} \epsilon_\mu^{\nu\rho\sigma} \text{Tr}(U^{-1} \partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U) \\ &\quad + \frac{ie}{8\pi^2} \epsilon_\mu^{\nu\rho\sigma} \partial_\nu [A_\rho \text{Tr} Q (U^{-1} \partial_\sigma U + \partial_\sigma U U^{-1})], \end{aligned} \quad (3.9)$$

and the expression for the derivative of U_K :

$$\begin{aligned} \partial_\mu U_K &= f_{,\mu} (-\sin f + i \cos f \tau_3) \\ &= i f_{,\mu} \tau_3 U_K. \end{aligned} \quad (3.10)$$

Clearly the first term ($B_{s\mu}$) vanishes, since f depends only on r and possibly t . The second term gives

$$\begin{aligned} B^0 &= -\frac{ie}{8\pi^2} \frac{1}{r^2 \sin \theta} \partial_\theta A_\phi \text{Tr} Q (U^{-1} U' + U' U^{-1}) \\ &= -\frac{ieg}{8\pi^2 r^2} \text{Tr} Q (2if' \tau_3) \\ &= \frac{f'}{8\pi^2 r^2} \\ \Rightarrow N_B &= \frac{1}{2\pi} [f]_0^\infty = 1, \end{aligned} \quad (3.11)$$

thus demonstrating that it is possible for a pure π^0 configuration to carry baryon number in the presence of the monopole [10]. The configuration (3.8) is called the radial kink [10].

iii) *Unwinding of the radial kink.*

Now let us consider the radial baryon flux of the kink. B^r vanishes, since f depends only on r and t . Therefore

$$\begin{aligned}
 B^r &= \frac{ie}{8\pi^2} \epsilon^{\nu\rho\sigma} \partial_\nu [A_\rho \text{Tr} Q (U_K^{-1} \partial_\sigma U_K + \partial_\sigma U_K U_K^{-1})] \\
 &= \frac{ie}{8\pi^2} \frac{1}{r^2 \sin \theta} \partial_\theta A_\phi \text{Tr} Q (U_K^{-1} \dot{U}_K + \dot{U}_K U_K^{-1}) \\
 &= -\frac{\dot{f}}{8\pi^2 r^2}.
 \end{aligned} \tag{3.12}$$

Thus we see that the radial flux of baryon number into the monopole core is $\frac{\dot{f}(0,t)}{2\pi}$. Whether or not $\dot{f}(0,t)$ can be non-zero depends on the boundary conditions at the monopole core. In the case of a grand unified monopole formed during an SU(5) or SO(10) phase transition for example, it is possible for baryon non-conserving boundary conditions to be placed, and hence for $\dot{f}(0,t) \neq 0$ [10].

Thus monopoles can catalyse skyrmion decay. We will now give a brief summary of the essential dynamical process.

Let us suppose that a standard nucleon field configuration is approaching the monopole and that the monopole will encounter some region in which the field configuration is non-trivial. Because of the suppression of charged particle wave functions near the monopole core, the nucleon field configuration will be forced into a radial kink. After passing the monopole the nucleon will regain a more conventional field configuration profile.

In passing through the core, the skyrmion must make the transition from one coordinate patch to another. Thus in the absence of baryon non-conserving boundary conditions, the field configuration must 'twist' as it passes over from one coordinate patch to another. This is in order to preserve baryon number. If, on the other hand, the boundary conditions do admit baryon decay, the skyrmion need not twist, but merely keeps its original profile, which in the new coordinate patch has baryon number zero.

This is the picture for monopole catalysis of skyrmion decay.

4. String catalysis of skyrmion decay.

We now turn to the case of a skyrmion interacting with a cosmic string. At first sight, we might expect some similarities with the monopole case, since the monopole has a semi-infinite Dirac string singularity, and we have an infinite string. However, this would be misleading; the Dirac string is a gauge singularity and can easily be removed by a more suitable description in terms of coordinate patches.

This is the crucial difference from the monopole setup. For a non-singular approach to the gauge field outside a monopole we needed to define two branches of the gauge field on two different coordinate patches, related by a non-trivial gauge transformation on the overlap. The cosmic string however, has a perfectly well defined gauge field without invoking coordinate patches. Thus the gauge field for a cosmic string exhibits no singularities, the additional term in (2.9b) is once more a total divergence, and baryon number is unchanged. Alternatively, if there are no gauge singularities, the equivalence classes of the soliton maps (2.4) are unchanged.

We will consider the equivalent of steps i) to iii) for the case of cosmic strings firstly by using a wire model for the string, and then by a more realistic vortex model.

I. WIRE MODEL

In grand unified models the string width is of the order of M^{-1} , where M is the grand unified mass. Thus, as a first approximation we take the string as a wire singularity [16] on the symmetry axis. In this case the string energy-momentum tensor is represented by the distributional form

$$S_0^0 = S_z^z = \mu \frac{\delta^{(2)}(r)}{2\pi r} ; S_r^r = S_\theta^\theta = 0.$$

Away from this singularity the gauge field is given by

$$A_\mu = -\frac{1}{e} \nabla_\mu \vartheta , \tag{4.1}$$

in cylindrical polar coordinates $\{\rho, \vartheta, z\}$.

In cylindrical polar coordinates, the static Klein-Gordon equation (3.7) reduces to

$$(\nabla_\mu \nabla^\mu - e^2 A_\mu^2) \varphi = - \left[\frac{1}{\rho} \partial_\rho \rho \partial_\rho + \partial_z^2 + \frac{1}{\rho^2} (\partial_\theta^2 - e^2 A_\theta^2) \right] \varphi = 0. \quad (4.2)$$

Here, rather like the monopole case, φ picks up extra "angular momentum" around the z -axis due to the presence of a non-zero A_θ . For the wire model, (4.2) gives

$$\left[\frac{1}{\rho} \partial_\rho \rho \partial_\rho + \partial_z^2 + \frac{1}{\rho^2} (\partial_\theta^2 - 1) \right] \varphi = 0, \quad (4.3)$$

which implies that the radial part of the wave equation for the lowest angular momentum eigenstate is

$$\rho \partial_\rho \rho \partial_\rho \varphi(\rho) = \varphi(\rho). \quad (4.4)$$

From this we deduce that φ must tend to zero as least as quickly as ρ near $\rho = 0$. Therefore, as in the monopole case, the wave functions of charged particles are suppressed near the core of the string, but those of uncharged particles need not be.

Unfortunately, it is now impossible for a radial kink to carry baryon number as a quick glance at (3.9,10) shows. B_s^μ is zero as before, hence

$$B^0 = \frac{ie}{8\pi^2} \epsilon^{\alpha\nu\rho r} \partial_\nu [A_\rho \text{Tr} Q (U^{-1} U' + U' U^{-1})]. \quad (4.5)$$

Translating (4.1) into spherical polars, we obtain

$$A_\mu = -\frac{1}{e} \nabla_\mu \phi, \quad (4.6)$$

which is a constant. We can now see immediately that $B^0 = 0$.

We have run into a problem here. Taking the wire approximation for a cosmic string leads to a suppression of the charged pion fields near the string. However, since a radial kink cannot carry baryon number on this case, we cannot have a deformation of the nucleon fields that would allow the skyrmion to approach the string core. Hence in the wire model of cosmic strings we do not get catalysis. Perhaps this problem is a result of approximating

the string core by a line. In order to be more physically realistic, we will consider a vortex model for the string – the Nielsen-Olesen vortex.

II. THE NIELSEN-OLESEN VORTEX

To illustrate the salient features of skyrmion catalysis by cosmic strings it is only necessary to consider an abelian theory. Thus we consider the Nielsen-Olesen vortex [17]. We discuss this in detail to facilitate the extension to the superconducting case. The Nielsen-Olesen string is a vortex solution to the lagrangian

$$\mathcal{L}[\phi, A_\mu] = D_\mu \phi^\dagger D^\mu \phi - \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{\lambda}{4} (\phi^\dagger \phi - \eta^2)^2, \quad (4.7)$$

where $D_\mu = \nabla_\mu + ieA_\mu$ is the usual gauge covariant derivative, and $\tilde{F}_{\mu\nu}$ the field strength associated with A_μ . However, we shall choose to express the field content in a slightly different manner, and one in which the physical degrees of freedom are made more manifest. We define the (real) fields X , χ and P_μ by

$$\phi(x^\alpha) = \eta X(x^\alpha) e^{i\chi(x^\alpha)} \quad (4.8a)$$

$$A_\mu(x^\alpha) = \frac{1}{e} [P_\mu(x^\alpha) - \nabla_\mu \chi(x^\alpha)] \quad (4.8b)$$

In terms of these new variables, the lagrangian becomes

$$\mathcal{L} = \eta^2 \nabla_\mu X \nabla^\mu X + \eta^2 X^2 P_\mu P^\mu - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda\eta^4}{4} (X^2 - 1)^2 \quad (4.9)$$

where $F_{\mu\nu}$ is the field strength associated with P_μ . The equations of motion are

$$\frac{\delta S}{\delta X} = 2\eta^2 \nabla_\mu \nabla^\mu X - 2\eta^2 P_\mu P^\mu X + \lambda\eta^4 X(X^2 - 1) = 0 \quad (4.10a)$$

$$\frac{\delta S}{\delta P_\nu} = -\frac{1}{e^2} \nabla_\mu F^{\mu\nu} - 2X^2 \eta^2 P^\nu = 0. \quad (4.10b)$$

We can see that a vacuum state is characterised by $X = 1$. However, this is not the only stable ground state which solves the equations of motion. Nielsen and Olesen showed that there exists a non-trivial stable ground state solution to the above equations of motion which has a vortex-like structure.

The Nielsen and Olesen vortex solution corresponds to an infinite, straight static string aligned with the z-axis. In this case, we can choose a gauge in which

$$\phi = \eta X(\rho) e^{i\theta} ; \quad P^\mu = P(\rho) \nabla^\mu \theta \quad (4.11)$$

in cylindrical polar coordinates. This string has winding number one. The equations for X and P_μ greatly simplify to

$$-X'' - \frac{X'}{\rho} + \frac{P^2 X}{\rho^2} + \frac{\lambda \eta^2}{2} X(X^2 - 1) = 0 \quad (4.12a)$$

$$-P'' + \frac{P'}{\rho} + 2e^2 \eta^2 X^2 P = 0 . \quad (4.12b)$$

These equations do not have any known analytic solutions, but asymptotic forms may be derived. It is not difficult to see that these are:

$$X \propto \rho ; \quad P = 1 + O(\rho^2) \quad \text{as } \rho \rightarrow 0, \quad (4.13a)$$

$$X \propto 1 - \rho^{-1/2} e^{-\sqrt{\lambda} \eta \rho} ; \quad P \propto \sqrt{\rho} e^{-\sqrt{2} e \eta \rho} \quad \text{as } \rho \rightarrow \infty. \quad (4.13b)$$

These will be sufficient for our purposes.

We now wish to use this Nielsen-Olesen vortex as a model for the cosmic string. Thus instead of using the wire form for the gauge field, (4.1), we use the expression for the Nielsen-Olesen gauge field, which is given by (4.8) and (4.11):

$$A_\mu = \frac{1}{e} (P(\rho) - 1) \nabla_\mu \vartheta . \quad (4.14)$$

This modifies the Klein-Gordon wave equation. The radial equation for the lowest angular momentum eigenstate, corresponding to (4.4), becomes

$$\rho \partial_\rho \rho \partial_\rho \varphi(\rho) = (P(\rho) - 1)^2 \varphi(\rho) . \quad (4.15)$$

From (4.13a), $P(\rho) = 1 + O(\rho^2)$ near $\rho = 0$, hence

$$\rho \partial_\rho \rho \partial_\rho \varphi(\rho) = O(\rho^4) \varphi(\rho),$$

allowing $\varphi \sim \text{const.}$ as $\rho \rightarrow 0$. Thus, on the scale of the core of the string, we need not have total suppression of charged particle wave functions.

Writing the analogue of (4.6) for the vortex A_μ in spherical polar coordinates gives

$$A_\mu = \frac{1}{e}(P(r \sin \theta) - 1)\nabla_\mu \phi. \quad (4.16)$$

As before, although slightly less trivially, substituting this into (4.5) shows the radial kink cannot carry baryon number. To see this, note that

$$\begin{aligned} B^0 &= \frac{ie}{8\pi^2} \epsilon^{0\nu\rho r} \partial_\nu [A_\rho \text{Tr} Q(U^{-1}U' + U'U^{-1})] \\ &= -\frac{ie}{8\pi^2} \frac{1}{r^2 \sin \theta} \partial_\theta A_\phi \text{Tr} Q(2if'\tau_3) \\ &= \frac{f'}{4\pi^2} \frac{\partial_\theta P(r \sin \theta)}{r^2 \sin \theta} \\ \Rightarrow N_B &= \frac{1}{2\pi} \int_0^\infty dr f' [P(r \sin \theta)]_0^\pi = 0. \end{aligned} \quad (4.17)$$

However, this is no longer critical for we can have all three pion fields approaching the core. Once the skyrmion is in contact with the core of the string, where the grand unified symmetry is essentially restored, the possibility of decay arises.

We will consider an unwinding process involving all three pion fields by making the simple Ansatz that the nucleon field configuration now depends on time:

$$U_N(\underline{x}, t) = \exp[iF(r, t)\hat{x} \cdot \underline{\tau}]. \quad (4.18)$$

The calculation of the baryon current for this field configuration is somewhat involved, and we relegate the details to an appendix.

The main result we need is the radial baryon current of the field configuration (4.18) which, from the appendix ((A15)), is given by

$$B^r = -\frac{\dot{F}}{4\pi^2 r^2} [P(\cos 2F - 1) + rP' \frac{\cos^2 \theta}{\sin \theta}]. \quad (4.19)$$

Integrating this over a sphere of radius r gives

$$\begin{aligned}
\int_{S^2} B^r d^2x &= \frac{-\dot{F}}{2\pi} \int_0^\pi d\theta \sin \theta [(\cos 2F - 1)P(r \sin \theta) + r \frac{\cos^2 \theta}{\sin \theta} P'(r \sin \theta)] \\
&= \frac{-\dot{F}}{2\pi} \int_0^\pi d\theta [(P \cos \theta)_{,\theta} + P \cos 2F \sin \theta] \\
&= \frac{\dot{F}}{\pi} - \frac{\dot{F} \cos 2F}{2\pi} \int_0^\pi d\theta \sin \theta P(r \sin \theta).
\end{aligned} \tag{4.20}$$

For small r , $P(r \sin \theta) = 1 + O(r^2)$ implies

$$\int_{S^2} B^r d^2x \simeq \frac{\dot{F}}{\pi} (1 - \cos 2F) \tag{4.21}$$

and hence that the flux of baryon number into the string core is $-\dot{F}(1 - \cos 2F)/\pi$.

Thus in the presence of baryon non-conserving boundary conditions, such as we would expect in the string core where the grand unified symmetry is unbroken, the skyrmion can unwind. Since $F(0) = \pi$ and $F(\infty) = 0$ for the standard nucleon field configuration (2.6), we expect that for an unwinding process F changes from π to 0 at the core of the skyrmion. And indeed

$$\begin{aligned}
\Delta N_B &= \int \dot{B}_N dt \\
&\simeq \int dt \dot{F} (1 - \cos 2F) / \pi \\
&= \frac{1}{\pi} \Delta [F - \frac{1}{2} \sin 2F] \\
&= -1
\end{aligned}$$

The residual field configuration is a topologically trivial excitation of the pion fields, and can therefore dissipate.

Thus strings can catalyse skyrmion decay. The picture however relies fundamentally on taking a vortex model for the string, i.e. one in which the string has a finite thickness. A model of the string with infinitesimal thickness (the wire model) gives no catalysis.

5. Skyrmion catalysis by superconducting cosmic strings.

In this section we consider the case of the skyrmion interacting with the superconducting cosmic string. Unlike its Nielsen-Olesen cousin, this has a long-range electromagnetic gauge field, hence we might expect some differences with the previous analysis. After all, one of the differences between the monopole and the Nielsen-Olesen vortex was the absence of long range interactions in the latter setup. However this is not the case as we will now show. First we consider the wire model for the superconducting string, then we derive a more realistic vortex model by solving the equations of motion near the core.

I. THE WIRE MODEL

Similar to the cosmic string case discussed previously, we can try taking the superconducting string to be a wire singularity on the symmetry axis. The long range electromagnetic gauge field is

$$A_z(\rho) = \frac{-I}{2\pi} \log(\rho/\rho_0), \quad (5.1)$$

where ρ_0 is the radius of the string, and I is the current in the string. Imposing (5.1) for $\rho > 0$ gives a wire model for the superconducting string.

Since we now have a long range electromagnetic field, we might expect some modifications of the previous analysis. Consider first the Klein-Gordon equation. In cylindrical polar coordinates, the Klein-Gordon equation (3.7) reduces to

$$\begin{aligned} (\nabla_\mu \nabla^\mu - e^2 A_\mu^2) \varphi &= - \left[\frac{1}{\rho} \partial_\rho \rho \partial_\rho + (\partial_z^2 - e^2 A_z^2) + \frac{1}{\rho^2} \partial_\theta^2 \right] \varphi \\ &= 0. \end{aligned} \quad (5.2)$$

Thus, similar to the monopole and cosmic string cases, φ picks up extra “angular momentum” due to the presence of a non-zero A_z . When we insert the form for A_z from (5.1) into (5.2) there is no analytic solution for φ . However, it is possible to show that charged particle wave functions are suppressed near the wire, but those of uncharged particles need not be.

In order to see if the radial kink can carry baryon number we need to consider the

corresponding form for A_μ in spherical polars,

$$\begin{aligned} A_r &= \cos \theta A_z(r \sin \theta) \\ A_\theta &= -r \sin \theta A_z(r \sin \theta) \\ A_\phi &= 0. \end{aligned} \tag{5.3}$$

As in the previous discussions, B_s^μ is zero for the radial kink. From (3.9) and (3.10) we obtain

$$B^0 = -\frac{ie}{8\pi^2} \epsilon^{0\mu\nu r} \partial_\mu A_\nu \text{Tr} Q(U_K^{-1} U'_K + U'_K U_K^{-1}), \tag{5.4}$$

and we see once again that the baryon number of the radial kink (3.8) must be zero. This is because there is no ϕ -component or ϕ -dependence in A_μ . Thus the previous discussion given for the ordinary cosmic string also applies to the case of superconducting cosmic strings: since the charged fields cannot approach the string core, and since a radial kink cannot carry baryon number, the nucleon cannot approach the core and unwind. In order to get catalysis, we have to look at a thick vortex model.

Incidentally, if we take a wire model of the $U(1) \times U(1)$ bosonic superconducting string, by setting the non-electromagnetic gauge field to the Nielsen-Olesen form (4.1), the behaviour of ϕ close to the wire is dominated by this field. The radial suppression of ϕ becomes exactly the same as for non-superconducting strings.

II. VORTEX MODEL FOR THE SUPERCONDUCTING STRING.

In order to obtain catalysis it seems necessary to consider a vortex model for the superconducting string. To obtain such a model, we consider the $U(1) \times U(1)'$ model of Witten [13]. Previously, only numerical solutions have been found [18], so we give details of our solution. Our discussion follows closely that for the Nielsen-Olesen string discussed in the previous section.

The lagrangian in this case is

$$\begin{aligned} \mathcal{L} &= D_\mu \phi^\dagger D^\mu \phi - \frac{1}{4} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} + D_\mu \sigma^\dagger D^\mu \sigma - \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \\ &\quad - \left[\frac{\lambda_\phi}{4} (\phi^\dagger \phi - \eta^2)^2 + (f|\phi|^2 - m^2)|\sigma|^2 + \frac{\lambda_\sigma}{4} |\sigma|^4 \right], \end{aligned} \tag{5.5}$$

where ϕ and σ are complex scalar fields; $\lambda_\sigma, \lambda_\phi$ and f are coupling constants;

$$\mathcal{D}_\mu \phi = \nabla_\mu \phi + ig C_\mu \phi$$

$$\mathcal{D}_\mu \sigma = \nabla_\mu \sigma + ie A_\mu \sigma,$$

C_μ and A_μ being abelian gauge fields carrying charges of g and e respectively, with $\tilde{G}_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ being the corresponding field strengths.

As in the Nielsen-Olesen situation we make a change of variables to clarify the physical content of the theory. Thus we set

$$\phi = R e^{i\theta} \quad (5.6a)$$

$$\sigma = S e^{i\chi} \quad (5.6b)$$

$$C_\mu = \frac{1}{g}(P_\mu - \nabla_\mu \theta) \quad (5.6c)$$

$$A_\mu = \frac{1}{e}(Q_\mu - \nabla_\mu \chi) \quad (5.6d)$$

In terms of these variables the new lagrangian becomes

$$\begin{aligned} \mathcal{L} = & \nabla_\mu R \nabla^\mu R + R^2 P_\mu P^\mu - \frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \nabla_\mu S \nabla^\mu S + S^2 Q_\mu Q^\mu - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \\ & - \left[\frac{\lambda_\phi}{4} (R^2 - \eta^2)^2 + (fR^2 - m^2) S^2 + \frac{\lambda_\sigma}{4} S^4 \right], \end{aligned} \quad (5.7)$$

where $G_{\mu\nu}$ and $F_{\mu\nu}$ are now the field strengths associated with P_μ and Q_μ respectively. The associated equations of motion are

$$\nabla_\mu \nabla^\mu R - R P_\mu P^\mu + \frac{\lambda_\phi}{2} R (R^2 - \eta^2) + f R S^2 = 0 \quad (5.8a)$$

$$\nabla_\mu \nabla^\mu S - S Q_\mu Q^\mu + \frac{\lambda_\sigma}{2} S^3 + (fR^2 - m^2) S = 0 \quad (5.8b)$$

$$\nabla_\mu G^{\mu\nu} + g^2 R^2 P^\nu = 0 \quad (5.8c)$$

$$\nabla_\mu F^{\mu\nu} + e^2 S^2 Q^\nu = 0 \quad (5.8d)$$

In analogy with the Nielsen-Olesen vortex, we solve equations (5.8) for a 'static' cylindrically symmetric superconducting string, i.e. one with constant current in the z -direction, $\chi \propto z$. We will write this constant of proportionality as ζ . This means that we

can choose a gauge in which

$$\begin{aligned} R &= R(\rho) , \quad S = S(\rho) , \\ P_\mu &= P(\rho)\nabla_\mu\vartheta , \quad Q_\mu = Q(\rho)\nabla_\mu z \end{aligned} \tag{5.9}$$

in cylindrical polar coordinates $\{\rho, z, \vartheta\}$. The equations of motion thus become

$$-R'' - \frac{R'}{\rho} + \frac{RP^2}{\rho^2} + \frac{\lambda_\phi}{2}R(R^2 - \eta^2) + fRS^2 = 0 \tag{5.10a}$$

$$-S'' - \frac{S'}{\rho} + SQ^2 + \frac{\lambda_\sigma}{2}S^3 + (fR^2 - m^2)S = 0 \tag{5.10b}$$

$$-P'' + \frac{P'}{\rho} + g^2R^2P = 0 \tag{5.10c}$$

$$-Q'' - \frac{Q'}{\rho} + e^2S^2Q = 0. \tag{5.10d}$$

We need to impose suitable boundary conditions for a vortex solution. These are

$$\begin{aligned} R &\rightarrow 0 \quad P \rightarrow 1 \quad Q \rightarrow \zeta \quad S \rightarrow S_0 \quad \text{as } r \rightarrow 0 \\ R &\rightarrow \eta \quad P \rightarrow 0 \quad S \rightarrow 0 \quad \text{as } r \rightarrow \infty \end{aligned} \tag{5.11}$$

where S_0 is given in terms of the other constants. These have been examined numerically in refs [18]. However, we need the analytic expressions for the solution near the origin. These can be seen to be

$$\begin{aligned} R &\propto \rho , \quad P = 1 + O(\rho^2) \\ S &= S_0 + O(\rho^2) , \quad Q = \zeta + O(\rho^2). \end{aligned} \tag{5.12}$$

Equation (5.10b) is satisfied by appropriate choice of the $O(\rho^2)$ coefficient of Q . Therefore we can read off the expression for the gauge fields in this vortex model as

$$\begin{aligned} C_\mu &= \frac{1}{g}(P(\rho) - 1)\nabla_\mu\theta \\ A_\mu &= \frac{1}{e}(Q(\rho) - \zeta)\nabla_\mu z. \end{aligned} \tag{5.13}$$

As with the Nielsen-Olesen vortex, the gauge fields (5.13) modify the Klein-Gordon equation (5.2). The radial equation now becomes

$$\begin{aligned}\frac{1}{\rho}\partial_\rho\rho\partial_\rho\varphi(\rho) &= [(Q(\rho) - \zeta)^2 + (P(\rho) - 1)^2/\rho^2]\varphi(\rho) \\ &= O(\rho^2)\varphi(\rho)\end{aligned}\tag{5.14}$$

which allows $\varphi(\rho) \rightarrow \text{const.}$ as $\rho \rightarrow 0$. Therefore, as with the Nielsen-Olesen vortex, on the scale of the core of the string, we do not have suppression of charged particle wave functions.

In order to calculate the baryon current we require the expression of A_μ in spherical polar coordinates. This is

$$\begin{aligned}A_r &= \frac{1}{e}(Q(r \sin \theta) - 1) \cos \theta \\ A_\theta &= -\frac{r}{e}(Q(r \sin \theta) - 1) \sin \theta \\ A_\phi &= 0.\end{aligned}\tag{5.15}$$

From (2.9b) we can see that $B^\mu = B_{No}^\mu$, the baryon current for the ordinary (Nielsen-Olesen) cosmic string, since the gauge field in (5.15) has no ϕ dependence or ϕ dependence. Therefore the radial kink cannot carry baryon number in this case. But, as with the Nielsen-Olesen vortex we will consider an unwinding of topological charge where all three pion fields approach the core of the string.

As before we use the time dependent nucleon Ansatz (4.18). The calculation of the baryon current proceeds in a similar fashion to the Nielsen-Olesen case. Since $B^\mu = B_{No}^\mu$, the radial baryon current is given by (4.19) as before. Thus we get the same baryon flux (4.21) as with the ordinary cosmic string,

$$\int_{S^2} B^r d^2x = \frac{\dot{F}}{\pi}(1 - \cos 2F),\tag{5.16}$$

giving the same baryon number flux as before, and hence

$$\Delta N_B = -1$$

as before.

Thus superconducting strings catalyse baryon decay. But, since we were forced to take a vortex model, i.e. a string with thickness, the process proceeds on the scale of the string core. Note once again, the important feature that on intermediate scales (between the core radius and the Compton wavelength of the nucleon), the Nielsen-Olesen gauge field dominates the behaviour of the ϕ -field. Thus any suppression of charged particle wave functions is the same as for ordinary non-superconducting cosmic strings.

6. Discussion and conclusions.

We have developed the argument of Callan and Witten for monopole catalysis of skyrmion decay in such a way that the effects of a topologically non-trivial gauge field are highlighted. We then explained the corresponding scenario for cosmic strings. We found that a wire model of the string was incompatible with catalysis, but that a vortex model admitted a catalysis scenario. This was also shown to be the case for superconducting strings. For the superconducting string we first had to derive the vortex solution, since previously this had only been found from numerical studies [18]. We did this by solving the string equations of motion in the core. With our vortex model we then showed that catalysis occurred.

These results support the following heuristic argument for the enhancement factor in the case of the monopole cross-section. Notice that the monopole argument was conducted exclusively within the approximation of the Dirac monopole; the only place the concept of a grand unified monopole occurred was in invoking baryon number non-conserving boundary conditions. By contrast, a thick string or vortex model was required in order to get catalysis to occur at all in the string picture. Thus in the monopole picture, the only scale we have is the skyrmion scale – giving an expected cross-section of the order of a strong interaction cross-section. On the other hand, the inescapability of the vortex model in the string case suggests that the reaction is occurring on the scale of the string radius, rather than the skyrmion radius, thus giving a grand unified cross-section.

So far our analysis suggests that the monopole catalysis cross-section is proportional to m^{-1} whilst the string and superconducting string cross-sections are proportional to M^{-1} , where m and M are the fermion and grand unified masses respectively. However, it is also possible to give the order of magnitude for the constant of proportionality. For the monopole we would expect the constant to be $O(1)$. This is because the process proceeds via the radial kink, and there is no suppression of the neutral pion wave function in the presence of the monopole. For the case of the string we can find the constant by an examination of the Klein-Gordon equation. For distances between m^{-1} and M^{-1} the equation for the charged particle wave function is (4.4). Thus, for these distances $\varphi \sim \rho$.

But for $\rho < M^{-1}$ the relevant wave equation is (4.15) and $\varphi \sim \text{const.}$ as $\rho \rightarrow 0$. To match solutions at $\rho = M$ we require that amplitude be proportional to m/M . Thus, we would expect this extra suppression factor for the string, giving a cross-section proportional to $(m/M)M^{-1}$. Similar arguments apply to the superconducting string case.

These results support the earlier calculations involving a quark/string scattering, that is, that there is no enhancement of the baryon decay cross-section for strings. Hence there will be no constraints on the cosmic string scenario from catalysis based on later time astrophysical processes. However, at earlier times in the evolution of the universe the string distribution was more concentrated. It is possible that this could have influenced the development of baryon asymmetry.

We realise these arguments are incomplete. A calculation of the cross-sections in each case (string and monopole) is required. This work is in progress. However, as it stands, the argument provides an elegant pictorial description of the skyrmion decay process. It shows clearly the difference between the monopole and string cases, and also readily obtains the superconducting string catalysis picture.

Appendix: Calculation of the radial baryon current.

In this appendix we will calculate the baryon current for the Ansatz nucleon field (4.18)

$$\begin{aligned} U_N(\underline{x}, t) &= \exp[iF(r, t)\hat{\underline{x}} \cdot \underline{\tau}] \\ &= \cos F + i \sin F \hat{\underline{x}} \cdot \underline{\tau} . \end{aligned} \quad (A1)$$

We will use spherical polar coordinates, in which the cosmic string gauge field takes the form

$$A_\mu = \frac{1}{e}(P(r \sin \theta) - 1)\nabla_\mu \phi . \quad (A2)$$

In deriving the baryon current

$$\begin{aligned} B_\mu &= \frac{1}{24\pi^2} \epsilon_\mu^{\nu\rho\sigma} \text{Tr} \left(U^{-1} \partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U \right) \quad (a) \\ &+ \frac{ie}{8\pi^2} \epsilon_\mu^{\nu\rho\sigma} \partial_\nu [A_\rho \text{Tr} Q (U^{-1} \partial_\sigma U + \partial_\sigma U U^{-1})] \quad (b), \end{aligned} \quad (A3)$$

the following expressions will be useful. Recall the identity

$$\tau_i \tau_j = \delta_{ij} + i \epsilon_{ijk} \tau_k . \quad (A4)$$

This implies

$$\tau_i \tau_j \tau_k = \tau_i \delta_{jk} - \tau_j \delta_{ik} + \tau_k \delta_{ij} + i \epsilon_{ijk} I_3 . \quad (A5)$$

We also need the expression for the derivative of U_N ,

$$\dot{U}_N = i \dot{F} \hat{\underline{x}} \cdot \underline{\tau} U_N \quad (A6a)$$

$$\partial_i U_N = i \hat{\underline{x}} \cdot \underline{\tau} F_{,i} U_N + \frac{i \sin F}{r} (\tau_i - \hat{x}_i \hat{\underline{x}} \cdot \underline{\tau}) . \quad (A6b)$$

Consider firstly term (a) in the baryon current (A3). From (A6) we may deduce

$$U_N^{-1} \partial_0 U_N = i \dot{F} \hat{\underline{x}} \cdot \underline{\tau} \quad (A7a)$$

$$U_N^{-1} \partial_i U_N = \frac{i \tau_j}{r} [r \hat{x}_j F_{,i} + \sin F \cos F (\delta_{ij} - \hat{x}_i \hat{x}_j) + \sin^2 F \epsilon_{ijk} \hat{x}_k] . \quad (A7b)$$

For notational convenience, we will write (A7b) as

$$U_N^{-1} \partial_i U_N = i \tau_j M_{ji} . \quad (A8)$$

Then the radial part of (a) is given by

$$\begin{aligned} (a)_r &= \frac{1}{24\pi^2} \epsilon_r^{\nu\rho\sigma} \text{Tr} \left(U_N^{-1} \partial_\nu U_N U_N^{-1} \partial_\rho U_N U_N^{-1} \partial_\sigma U_N \right) \\ &= \frac{-i}{24\pi^2} (3!) \epsilon_r^{0\theta\phi} \text{Tr} (\dot{F} \hat{x}_i \tau_i \tau_j M_{j\theta} \tau_k M_{k\phi}) \\ &= \frac{i\dot{F}}{4\pi^2 r^2 \sin \theta} \hat{x}_i M_{j\theta} M_{k\phi} \text{Tr} (\tau_i \tau_j \tau_k) \\ &= -\frac{\dot{F}}{2\pi^2} (M_{\theta\theta} M_{\phi\phi} - M_{\theta\phi}^2) \\ &= -\frac{\dot{F}}{2\pi^2} \frac{\sin^2 F}{r^2} . \end{aligned} \quad (A9)$$

In order to calculate the radial current arising from term (b), we note that

$$\begin{aligned} Q &= \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \\ \Rightarrow \quad \text{Tr} Q \tau_i &= \delta_{i3} . \end{aligned} \quad (A10)$$

Thus

$$\begin{aligned} \text{Tr} Q (U_N^{-1} \dot{U}_N + \dot{U}_N U_N^{-1}) &= 2i \dot{F} \hat{x}_i \text{Tr} Q \tau_i \\ &= 2i \dot{F} \hat{x}_3 \end{aligned} \quad (A11)$$

and

$$\begin{aligned} \text{Tr} Q (U_N^{-1} \partial_i U_N + \partial_i U_N U_N^{-1}) &= \text{Tr} Q (2i \hat{x}_i \sin F_{,i} + 2i \sin F \cos F (\delta_{ij} - \hat{x}_i \hat{x}_j) \frac{\tau_j}{r}) \\ &= 2i \hat{x}_3 F_{,i} + 2i \sin F \cos F (\delta_{i3} - \hat{x}_i \hat{x}_3) / r \end{aligned} \quad (A12)$$

imply

$$\text{Tr} Q (U_N^{-1} \dot{U}_N + \dot{U}_N U_N^{-1}) = 2i \dot{F} \cos \theta \quad (A13a)$$

$$\text{Tr} Q (U_N^{-1} U_{N,\theta} + U_{N,\theta} U_N^{-1}) = -2i \sin F \cos F \sin \theta . \quad (A13b)$$

Therefore the contribution to the radial baryon current from term (b) becomes

$$\begin{aligned}
(b)_r &= \frac{ie}{8\pi^2} \epsilon_r^{\nu\rho\sigma} \partial_\nu [A_\rho \text{Tr} Q (U^{-1} \partial_\sigma U + \partial_\sigma U U^{-1})] \\
&= \frac{ie}{8\pi^2} \epsilon_r^{0\phi\theta} \left\{ \partial_0 [-2i \sin F \cos F \sin \theta A_\phi] - \partial_\theta [2i \dot{F} \cos \theta A_\phi] \right\} \\
&= -\frac{1}{4\pi^2} \frac{\dot{F}}{r^2 \sin \theta} \left\{ [-\cos 2F \sin \theta (P(r \sin \theta) - 1)] - \partial_\theta [\cos \theta (P(r \sin \theta) - 1)] \right\} \\
&= -\frac{\dot{F}}{4\pi^2 r^2} \left\{ -\cos 2F (P - 1) + (P - 1) - r P' \frac{\cos^2 \theta}{\sin \theta} \right\} \tag{A14}
\end{aligned}$$

Combining the two expressions (A9) and (A14) gives the total radial baryon current

$$\begin{aligned}
B_r &= -\frac{\dot{F} \sin^2 F}{2\pi^2 r^2} - \frac{\dot{F}}{4\pi^2 r^2} \left\{ -\cos 2F (P - 1) + (P - 1) - r P' \frac{\cos^2 \theta}{\sin \theta} \right\} \\
&= \frac{\dot{F}}{4\pi^2 r^2} \left[P(\cos 2F - 1) + r P' \frac{\cos^2 \theta}{\sin \theta} \right]. \tag{A15}
\end{aligned}$$

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